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MONTHLY WEATHER REVIEW

JAMES E. CASKEY, JR., Editor

Volume 83
Number 11

NOVEMBER 1955

Closed January 15, 1956
Issued February 15, 1956

THE ECONOMIC UTILITY OF WEATHER FORECASTS

J. C. THOMPSON

Weather Bureau Airport Station, Los Angeles, Calif.[1]

and

G. W. BRIER

U. S. Weather Bureau, Washington, D. C.

[Manuscript received January 17, 1955; revised November 14, 1955]

ABSTRACT

The economic factors involved in the use of weather forecasts are discussed, and procedures for analyzing the economic utility of both probability and categorical forecasts are derived. Some of the considerations involved in making public forecasting decisions are presented, and expressions are suggested for assessing the economic utility of public forecasts. The relationships between these measures of economic usefulness and certain formulae frequently used to assess forecasting accuracy are also pointed out.

1. INTRODUCTION

One of the problems which, from time to time, has faced the meteorologist is the need for measuring his ability to predict the state of the atmosphere in order that the value of his scientific effort may be demonstrated. This requirement has resulted in a number of studies aimed at devising "verification systems" whose principal purpose has been to assess the accuracy of the prediction. Since this requires that such accuracy be defined, often quite arbitrarily, it is not an uncommon experience to find that a group of forecasts which may show a high verification score need not necessarily be economically useful predictions [1].

It is therefore the purpose of this paper to present a method of analysis designed to measure the economic utility of the forecast, and to suggest a verification procedure based upon the operational risks involved in taking protective measures against adverse weather. In this way, the definition of forecasting accuracy may be made synonomous with economic usefulness, thus overcoming the difficulty.

[1] Present address: U. S. Weather Bureau, Washington, D. C.

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2. AN ECONOMIC DECISION CRITERION

Consider the general case of a potential user of a weather forecast faced with the problem of deciding whether or not to take protective measures against a certain adverse weather element, W. In general, he should take such protective measures if, in the long run, some economic gain will be realized; otherwise no protective measures should be taken. In order to derive a criterion for making this decision, the following terms are defined:

G_p = Total expected gain for $N=(f_w+f_{nw})$ days of operation if protective measures are taken every day,

G_{np} = Total expected gain for N days if no protective measures are taken,

C = Cost of protection each day that protective measures are taken,

L = Loss suffered each day that adverse weather occurs and no protective measures have been taken,

T = Average daily net operating income exclusive of the cost of protection (C) which may have been taken, or the loss (L) which may have been suffered,

f_w = Frequency of adverse weather,

f_{nw} = Frequency of favorable weather.

If, now, protective measures are taken every day, the total gain will be the daily net operating income minus the daily cost of protection, both times N, the number of days of operation. Thus,

$$G_p=(T-C)N. \tag{1}$$

If no protective measures are taken, it will be apparent that,

$$G_{np}=(T-L)f_w+Tf_{nw}. \tag{2}$$

The total gain to maximize the profit on the entire operation should be as large as possible; thus protective measures should be taken whenever $G_p>G_{np}$ or, from (1) and (2), protection would be required if

$$(T-C)N>(T-L)f_w+Tf_{nw}.$$

This reduces to

$$\frac{f_w}{N}>\frac{C}{L}.$$

The left-hand side of this inequality defines P, the "probability" of adverse weather [2]. In a similar manner it may be shown that protective measures should not be taken if $P<C/L$, and the total gain would be equal whether or not such measures were taken if $P=C/L$. Thus the criterion for making a decision to protect or not protect may be expressed

$$P\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}\frac{C}{L}\begin{cases}\text{Protect}\\ \text{Either course}\\ \text{Not protect}\end{cases} \tag{3}$$

The value $P=C/L$ therefore represents a critical ratio, above which protection should be provided, and below which it should not. It is interesting to note that for C, L, and T, as defined here, the last drops out and need not be considered in making the decision. Alternative, but generally more complex, expressions may be derived by defining these terms in a different manner, e. g., Gringorton [3].

3. THE ECONOMICS OF WEATHER FORECASTS

The results of a series of N categorical forecasts are presented in a generalized form in table 1. Here W and No W are defined as the occurrence and non-occurrence, respectively, of an operationally critical weather event, and a, b, c, d, represent the frequencies in the indicated boxes in the table.

From table 1, the climatological probability, P_c, of observing a critical weather event is given by

$$P_c=\frac{c+d}{N}. \tag{4}$$

From equation (3) it is seen that, if $P_c>C/L$ and climatological expectancy is used as a basis for the decision,

TABLE 1.—*Generalized two-class forecast—observed contingency table.*

		FORECAST		
		No W	W	Total
OBSERVED	No W	a	b	$a+b$
	W	c	d	$c+d$
	Total	$a+c$	$b+d$	$N=a+b+c+d$

protection should be provided every day. In this case there is no loss, and the total expense for the operation E_p, for N days is given by

$$E_p=CN \tag{5}$$

When $P_c<C/L$, no protective measures should be taken, and a loss will be suffered every day that W occurs. The total expense for the operation in this case is

$$E_{p'}=L(c+d) \tag{6}$$

If, on the other hand, the decision is based on the forecasts in table 1, the total expense for the operation, E_f, will be due to the cost of protection whenever W is forecast plus the loss due to missed predictions. Thus

$$E_f=C(b+d)+Lc. \tag{7}$$

As a matter of interest, it should be noted that this expression could be used in assessing the economic utility of a weather forecast [4, 5]. It suffers, however, from two limitations: (a) It provides no inherent frame of reference by which the utility of the forecast is compared with another method of making a decision; and (b) since it measures the expense of the operation, the best forecast is obtained with the lowest value of E_f. If, however, it is noted that equations (5) and (6) give the operational expense for decisions based on climatology, and if the operational expense resulting from the use of the forecasts is compared with these expressions in the proper way, the economic saving over climatology can be obtained, thus providing a more useful measure of forecast utility.

When $P_c\geq C/L$, the economic saving over climatology, S_1, per unit forecast per unit of loss is given by

$$S_1=\frac{E_p-E_f}{NL}.$$

Using equations (5) and (7), the above expression may be simplified in the following form and used to measure the economic usefulness of the forecast:

$$S_1=\frac{a+c}{N}\left(\frac{C}{L}-\frac{c}{a+c}\right)\quad (\text{Use when } P_c\geq C/L). \tag{8}$$

In a similar manner, when $P_c\leq C/L$, the economic saving over climatology, S_2, is

$$S_2=\frac{E_{p'}-E_f}{NL}$$

or,

$$(9)\qquad S_2=\frac{a+c}{N}\left(\frac{C}{L}-\frac{c}{a+c}\right)+\left(P_c-\frac{C}{L}\right)\ \text{(Use when } P_c\leq C/L\text{)}.$$

Equations (8) and (9) now provide a quantitative measure of the value of any categorical weather prediction where the relative economic risks, C/L, are specified. If the loss, L, is expressed in dollars, the quantity S_1 or S_2 gives the number of dollars saved over climatology for each forecast issued.

4. ECONOMIC GAINS FOR PROBABILITY FORECASTS

If verification data are available for a series of probability forecasts, it is possible to compute the economic value of the forecasts for any given value of the economic risks, C/L, using equations (8) and (9). In this case, the forecasts may be placed in a contingency table similar to table 1, but with the decision to forecast W or No W being based upon the value of C/L (and therefore of P) selected:

TABLE 2.—*Generalized contingency table for use with probability forecasts*

		FORECAST PROBABILITY		
		$P\leq C/L$	$P>C/L$	Total
OBSERVED	No W	a	b	$a+b$
	W	c	d	$c+d$
	Total	$a+c$	$b+d$	$N=a+b+c+d$

Where it is desired to compute the economic saving for a number of values of C/L, it is convenient to make the computations by cumulating the forecast and observed critical weather frequencies, thus permitting carrying out the computations in a single table. Table 3 shows an example of such computations using a series of experimental forecasts of the probability of occurrence of minimum temperatures 32° F. or less for a selected station during the winter months of 1950–51. The climatological relative frequency, P_c, from the observed series was 0.64.

TABLE 3.—*Verification data and computation of economic savings for probability forecasts of minimum temperatures at a selected station for December 1950, January and February 1951. (Data for one day missing)*

Forecast probability	No. of forecasts	Accumulated forecasts	Accumulated occurrences of $T\leq 32°$	$\frac{a+c}{N}$	$\frac{c}{a+c}$	(P_c-C/L)	S
0.05	11	11	1	0.1236	0.0909	----------	−0.01
.15	9	20	1	.2247	.0500	----------	.02
.25	6	26	2	.2921	.0769	----------	.05
.35	1	27	2	.3034	.0741	----------	.08
.45	5	32	6	.3596	.1875	----------	.09
.55	10	42	15	.4719	.3571	----------	.09
.65	6	48	18	.5393	.3750	−0.01	.14
.75	9	57	27	.6404	.4737	−.11	.07
.85	6	63	31	.7079	.4921	−.21	.04
.92	9	72	40	.8090	.5556	−.28	.01
.97	17	89	57	1.0000	.6404	−.33	.00

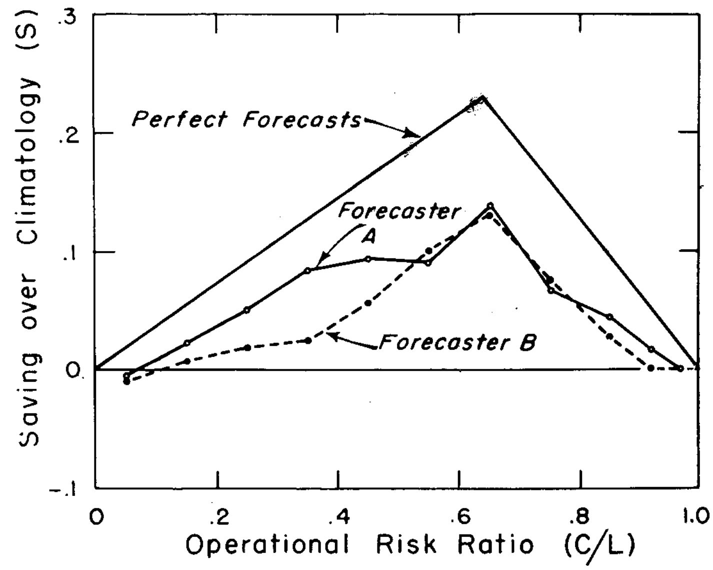

FIGURE 1.—Saving over climatology for a series of experimental probability predictions made by two different forecasters, A and B. Forecasts were made for temperatures 32° F. or lower at a selected station during the winter months of 1950–51.

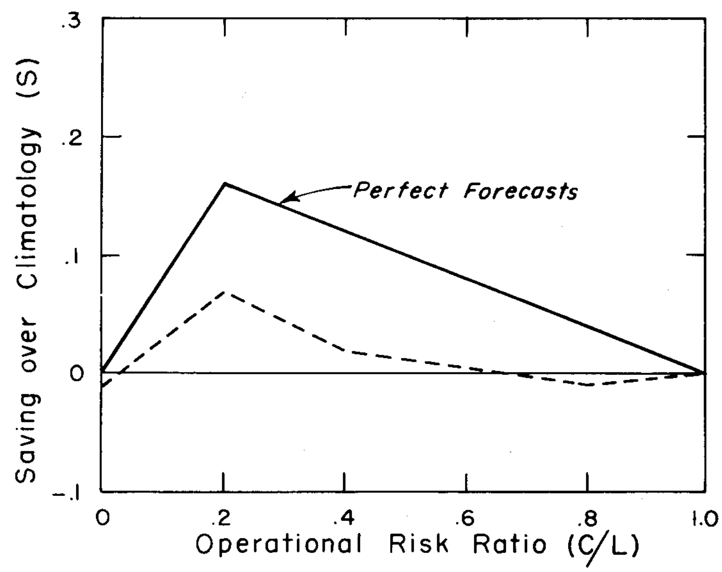

FIGURE 2.—Saving over climatology for a series of probability predictions discussed by Williams [6]. Forecasts were made for precipitation at Salt Lake City.

Figure 1 is a graph of these data (Forecaster A) and data for a similar set of probability forecasts made by another individual (Forecaster B). The maximum savings over climatology for perfect forecasts are also shown.

Figure 2 shows the savings for a group of precipitation forecasts discussed by Williams [6]. It will be noted that the utility of these forecasts would be limited to operations whose relative economic risks would range between 0.05 and 0.6, approximately.

It is also of interest to note that the maximum economic saving occurs where the ratio C/L is numerically equal to the climatological probability of adverse weather. An inspection of equations (8) and (9) will show that S_1 has a positive, and S_2 a negative, linear relationship to C/L. Since the useful portion of S_1 is limited to the segment between $C/L=0$ and its intersection with S_2 at $C/L=P_c$, while the useful part of S_2 ranges between the latter value and $C/L=1$, the maximum economic gain for any given set of forecasts will occur where the relative economic risks are equal to the climatological expectancy. This suggests that weather forecasts in general will have the greatest chance of being useful, as defined in this paper, when made for operations of this kind.

5. PUBLIC FORECASTING DECISIONS

Although, as shown by Ogarawa [7], the economic advantages inherent in the use of probability forecasts are undeniable, there are several practical difficulties which appear to inhibit the issuance of probability estimates for general public use—at least for the present. Among these difficulties are the lack of experience on the part of the forecasters in issuing probability forecasts, the need for public education regarding their use, and a number of technical difficulties arising from the necessity for simplifying a somewhat complex concept without invalidating certain basic principles. Furthermore, it seems likely that even if a simple probability forecasting system were devised for general use, a large percentage of the public, following an economically unsound but mentally less fatiguing policy of letting the forecaster make their operating decisions for them, would ignore the probability aspects of the forecast and continue to use the prediction as a categorical statement. It would, therefore, be of interest to present some of the general principles which relate categorical forecasts to the overall economic considerations which govern their optimum use.

From the previous discussion, it is evident that a categorical forecast should be based upon the nature of the economic risks, C/L, applicable to the operation for which the forecast is being issued. This ratio for a single forecast user may be obtained from an analysis of the operation. It is interesting to speculate, however, concerning the optimum value of C/L for a series of public forecasts where a wide range of operations is involved. Two such values are worth mentioning here.

In order to make economic sense, the ratio C/L can be shown to have a total range between zero and unity. Consider, for example, the possibility that $C/L>1$. In this case, the cost of protection, C, would exceed the loss, L, and it would obviously be uneconomic to consider protecting against adverse weather at all. In a similar way, it will be seen that negative values of C/L are economically meaningless.

Since $0\leq C/L\leq 1$, if it were assumed that the relative risks involved in making decisions for the usual day-to-day public forecasts would include this entire range, and that all values of the ratio C/L were equally important, then it might be suggested that the categorical prediction of greatest overall public usefulness would be obtained if the decision to forecast critical weather were made when this ratio is near the middle of the range, i. e., when $C/L=.50$. Inserting this value in equations (8) and (9) gives the very simple expressions:

(10) $$S_{1\,(C/L=.5)}=\frac{a-c}{2N} \qquad \text{(Use when } P_c\geq .5\text{)}$$

(11) $$S_{2\,(C/L=.5)}=\frac{d-b}{2N} \qquad \text{(Use when } P_c\leq .5\text{)}$$

Thus, if the forecaster bases his decision to forecast rain or no-rain, say, on the basis that the chances are greater or less than even, i. e., that the probability of rain is greater or less than .50, equation (3) states that such forecasts have been designed for operations where the value of C/L is likewise near .50. Equations (10) or (11) then provide a quantitative measure of the economic value of the predictions. As a rule (but not necessarily), selecting the decision criterion at the .50 probability level will result in the prediction of adverse and favorable weather with about the same frequencies as they occur, i. e., in table 1 $(b+d)$ will be approximately equal to $(c+d)$. This is equivalent, in the usual forecast terminology, to saying that adverse weather will be neither "under-forecast" nor "over-forecast," a procedure which is considered by many forecasters to be desirable, e. g., Schmidt [8].

Another interesting result is obtained if the ratio C/L is assumed to be near the climatological probability of adverse weather where, as pointed out earlier, the maximum economic gain will be realized. Accordingly, if P_c as defined in equation (4) is substituted for C/L in equations (8) or (9), the economic saving over climatology will be given by

(12) $$S_{(C/L=P_c)}=\frac{ad-bc}{N^2}.$$

That it may be desirable to use the climatological expectancy as a basis for the decision to predict unfavorable weather is suggested by the fact that both agriculture and industry as well as the living habits of the people, tend to become adjusted to the normal climatic expectancy of various weather elements. In the San Joaquin Valley of California, for example, a thirty-five million dollar raisin crop is spread under the open sky for a period of four to six weeks on the strong climatic expectancy that no rain of consequence will occur during the early fall months. If rain does occur unexpectedly, however, the loss is so great, and the cost of covering or otherwise protecting the exposed crop so small, that the operational risk, C/L, is reduced to a very low value. This, in turn, means that the decision to issue rain warnings to the growers should be made at a rather low probability level—perhaps near the climatological expectancy.

Similar examples for the public as a whole might be suggested, especially in the cases of destructively severe weather—hurricanes, tornadoes, blizzards, and the like—where the climatic frequency is also rather low. In these cases the general result of making the forecasting decision at the climatological probability level will be to "over-forecast" severe weather, i. e., in table 1 $(b+d)$ will usually be considerably greater than $(c+d)$.

It should be pointed out that every group of categorical forecasts is associated, explicitly or implicitly, with some value, or values, of the ratio C/L. For the most part, these factors are only vaguely incorporated in public forecasts at present. The values of C/L given here are suggested as interesting and perhaps, for some purposes, fairly realistic values.

6. ECONOMIC GAINS FOR PUBLIC FORECASTS

In order to give some idea of the order of magnitude of the economic gains which may result from the use of categorically issued public forecasts, the set of temperature forecasts in table 4 is reproduced from a previous paper [1]. These are the official forecasts for the same period as the experimental probability forecasts of table 3 and figure 1.

For $C/L=.50$ (since $P_c=\frac{57}{90}=.63>.50$, equation (10) is used):

$$S_{1(C/L=.5)}=\frac{27-6}{180}=.117$$

For $C/L=P_c$, using equation (12):

$$S_{(C/L=P_c)}=\frac{27\times 51-6\times 6}{90^2}=.166.$$

These values indicate that, for this set of predictions, savings of 11.7 and 16.6 cents per forecast, for each potential dollar of loss, would be obtained where the operational risks were near .50 and .63, respectively. As was shown in the previous paper [1] where these data were used, these forecasts would only be useful where $.18<C/L<.90$, approximately. For economic risks outside this range, values of S_1 and S_2 will be found to be negative.

TABLE 4.—*Minimum temperature forecast verifications at a selected station for the months of December, 1950; January and February, 1951*

		FORECAST		
		>32	≤32	Total
OBSERVED	>32	27	6	33
	≤32	6	51	57
	Total	33	57	90

7. RELATION TO OTHER MEASURES OF FORECASTING SKILL

As a matter of interest, it may be worth noting that equations (10), (11), and (12) are related to the percentage of correct forecasts A as follows:

$$S_{1(C/L=.5)}=\frac{1}{2}(A-P_c)$$

$$S_{2(C/L=.5)}=\frac{1}{2}[A-(1-P_c)]$$

$$S_{(C/L=P_c)}=\frac{1}{2}\left(A-\frac{E_c}{N}\right)$$

where $A=\frac{a+d}{N}$ and $E_c=\frac{(a+b)(a+c)+(c+d)(b+d)}{N}=$ number of forecasts expected correct by chance as determined by the marginal totals of table 1.

Equations (10), (11), and (12) are related to the conventional skill score S_c in the following manner:

$$S_{1(C/L=.5)}=\frac{1}{2}\left[\frac{E_c}{N}(1-S_c)+(S_c-P_c)\right]$$

$$S_{2(C/L=.5)}=\frac{1}{2}\left[\frac{E_c}{N}(1-S_c)+S_c-(1-P_c)\right]$$

$$S_{(C/L=P_c)}=\frac{1}{2}\left[S_c\left(1-\frac{E_c}{N}\right)\right]$$

where $S_c=\frac{(a+d)-E_c}{N-E_c}$.

Certain other alternative relationships between these variables may be shown with a little algebraic manipulation.

ACKNOWLEDGMENT

The authors wish to express their appreciation for suggestions received during discussions with Mr. R. A. Allen and Mr. E. M. Vernon.

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